IMPROVING PERFORMANCE OF DIRECTION-OF-ARRIVAL (DOA) ESTIMATION USING SIGNAL PROCESSING ON GRAPHS (SPG)

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DOA estimation from data collected from an array of sensors enhances our situational awareness, and the single-snapshot case presents a unique set of challenges and opportunities for research using the emerging field of signal processing on graphs (SPG)

> **Most studied DOA estimation method**: Bartlett method

> **Limitation of Bartlett method**: large estimation bias

> **Conventional DSP framework**: data lies on a regular uniform grid

> **SPG framework**: data lies on vertices of an irregular graph domain

> **Graph consists of**: vertices and edges

Which graph structure in the SPG framework will replicate the Bartlett method?

Can we add an edge to this graph that will improve the performance of the Bartlett method?

The scope of our studied is narrowed to the following:

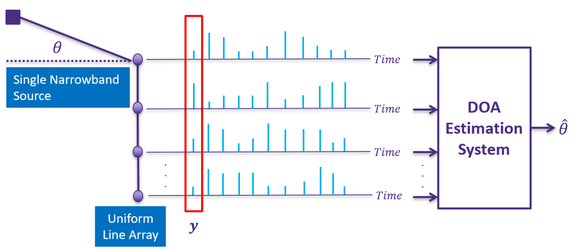
> Single far-field and narrowband source signal

> Uniform line array of sensors

> Known inter-element spacing between sensors

SINGLE-SNAPSHOT DOA ESTIMATION

> **Goal:** estimate the DOA using only a single-snapshot of recorded data

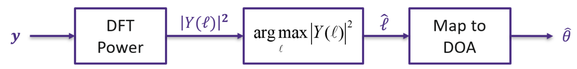


> **Important Variables:**

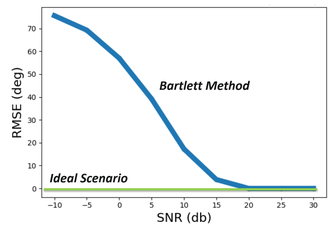
$\boldsymbol{y}$	Single-snapshot of recorded array data (source + noise)
$\hat{\theta}$	Estimate of DOA (in degrees)
θ	True DOA (in degrees)
N	Number of array element
d	Inter-element spacing between array elements
f_o	Frequency of source signal (in Hz)
c	Speed of sound (in m/sec)

DOA ESTIMATION WITH BARTLETT METHOD

> Estimates DOA by taking the Discrete Fourier Transform (DFT) of the received single snapshot of data and finding the **frequency bin with maximum power**

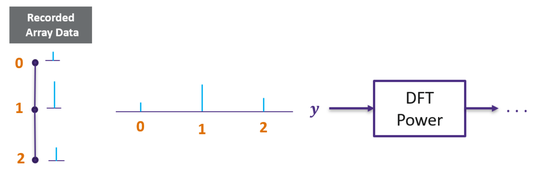


> Estimation performance suffers from **large bias** with increasing noise



DOA Estimation Performance of Bartlett Method. Plot of typical bias of DOA estimates for Bartlett method (in blue), measured using a root-mean-square error (RMSE), as a function of signal-to-noise (SNR) ratio. These measurements are generated using 2000 trials of data simulated for an array of 3 sensors with sources transmitted at $-75^\circ, 0^\circ, +75^\circ$.

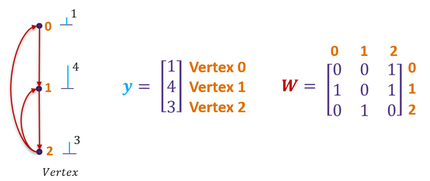
> **Underlying assumption in process:** data **y** lies on a regular uniform grid of points



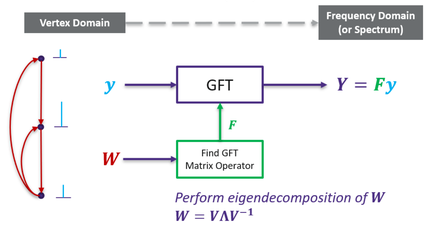
SPG FRAMEWORK

> **Data modeled** on a graph domain (with vertices and edges) to form a graph signal

> **Graph Signal** = *Measured Data* + *Underlying Graph*

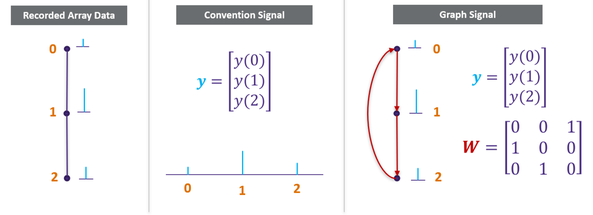


> **One tool for data processing in SPG**: Graph Fourier Transform (GFT), which extends the conventional Discrete Fourier Transform (DFT)



GRAPH STRUCTURE FOR BARTLETT METHOD

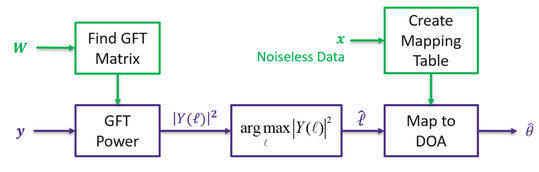
> The underlying graph structure for the graph signal that replicates the Bartlett method is a **directed cycle graph**



> Entries in GFT matrix operator **F** for a directed cycle graph turn out to be values for a **complex exponential signal**, which are the same values for the conventional DFT operation

> DOA can be estimated from a graph signal using a modified process that replicates the Bartlett method in the graph domain

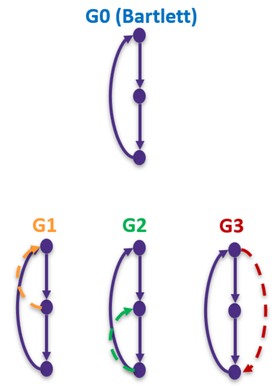
> **Modifications to the Bartlett method** processing system from conventional signal processing are highlighted below in green:



USING OTHER GRAPH STRUCTURES

> SPG offers a framework to explore **other data processing domains** by simply modifying the graph structure

> **Our focus:** study and compare estimation performance for data processed on graphs with only 1 additional directed edge



FINDING BEST EDGE TO ADD TO GRAPH

> **Criteria for best edge**: lower root-mean square error (RMSE) than the Bartlett method, where RMSE (in degrees) is given by the following

$$RMSE = \sqrt{E\left[\frac{1}{K}\sum_{k=1}^{K}\left(\hat{\theta}_k - \theta_k\right)^2\right]}$$

Estimate DOA; True DOA; Average of # of Trials; Average over # of Test DOAs

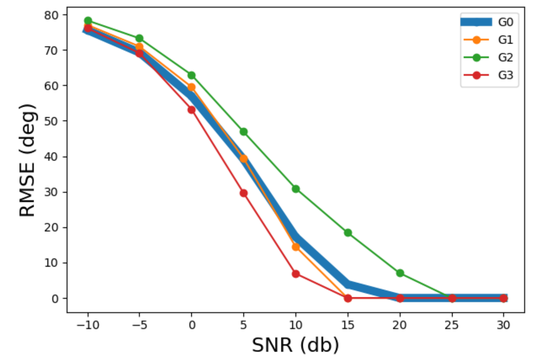
> **Other testing parameters:**

$\theta = [-75^\circ, 0^\circ, 75^\circ]$	True DOA (in degrees)
$N = 3$	Number of array elements
$d = 0.8$ m	Inter-element spacing between array elements
$f_o = 400$ Hz	Frequency of source signal (in Hz)
$c = 1500$ m/sec	Speed of sound (in m/sec)
$-10:5:30$ dB	Signal-to-noise ratios (in dB)
2000	Number of trials for data single-snapshots

RESULTS

> **Graph structure G3** had the best improvement in RMSE over all other graphs when compared to graph G0 (the Bartlett method)

> **Implication:** we can improve DOA estimation over conventional Bartlett method by adding an edge to the replicated graph **G0**



DOA Estimation Performance Comparison. Root-mean-square error (RMSE), as a function of signal-to-noise (SNR) ratio, is plotted for 4 total graph structures. These measurements are generated using 2000 trials of data simulated for an array of 3 sensors with sources transmitted at $-75^\circ, 0^\circ, +75^\circ$.

FUTURE WORK

> **Identify unique properties** of graph G3 over other graph structures

> **Extend this study** to a higher number of nodes N

> **Extend this work** to other array structures like non-uniform line and nonlinear arrays

REFERENCES

> Sandryhaila, Aliaksei, and Jose MF Moura. "Big data analysis with signal processing on graphs." *IEEE Signal Processing Magazine* 31, no. 5 (2014): 80-90.

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