# Towards Intrinsic Reference Tracking for Legged Locomotion


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## Introduction

Trajectory tracking (comparing actual motion to desired motion) allows the robust realization of predefined behaviors in real-world robots. For legged robots, every footfall corresponds to a sudden change in velocity, causing a large tracking error when the desired and actual trajectories impact at different times. We seek to construct a locally continuous intrinsic distance measure for fully-actuated mechanicals systems undergoing inelastic collisions.

## Tracking a perfectly elastic bouncing ball through impacts

1. The bouncing ball is governed by two sets of dynamics, one being gravity and the other the reset that occurs at a height $h$ of 0. The **reset law** $\boldsymbol{R}: \boldsymbol{TC} \to \boldsymbol{TC}$ maps velocity pre-impact velocity to post-impact velocity,

$$R(h,\dot{h}) = (h, -\dot{h}).$$

2. $\boldsymbol{P}: \tilde{\boldsymbol{C}} \to \boldsymbol{C}$ defines a map that takes trajectories generated in the extended space $\tilde{C}$ to trajectories that undergo impacts [1].

$$P(h) = \begin{cases} h, & h \geq 0 \\ -h, & h < 0. \end{cases}$$

3. Defining the dynamics on $\tilde{C}$ such that a trajectories in $\tilde{C}$ map to trajectories in $C$ under $P$ [2] yields a continuous switched system.

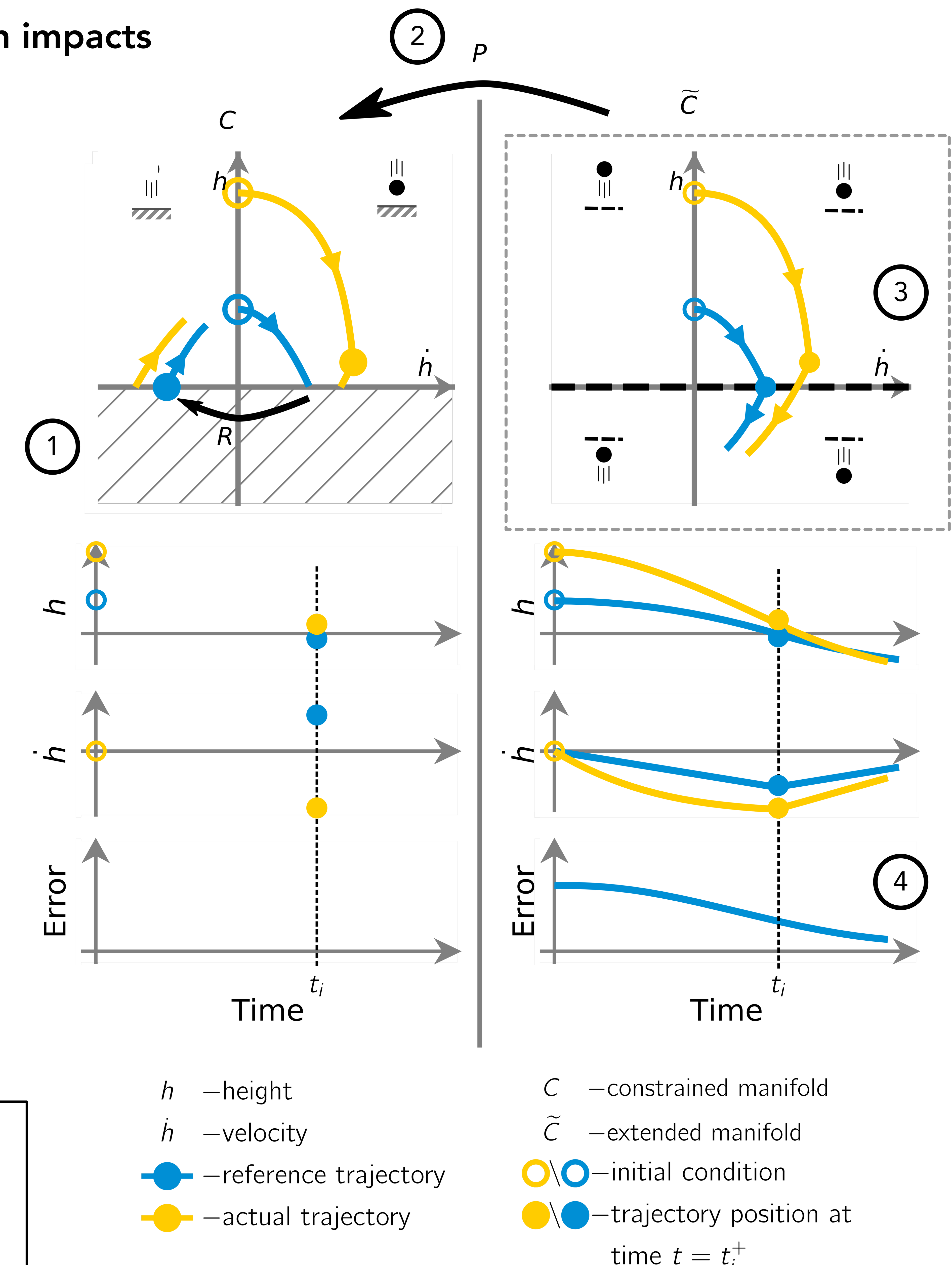
Let $\tilde{r}(t)$ be the reference trajectory in $\tilde{C}$, then

$$\ddot{\tilde{r}}(t) = F(h(t), \dot{h}(t)) = \begin{cases} -g, & h(t) \geq 0 \\ g, & h(t) < 0. \end{cases}$$

4. Using results from geometric control [3], the reference trajectory can be tracked on $\tilde{C}$, implying tracking on $C$ away from impacts.

Let $\tilde{q}(t)$ be the actual trajectory in $\tilde{C}$, then

$$\begin{aligned} \ddot{\tilde{q}}(t) &= F_c\left(\tilde{q}(t), \dot{\tilde{q}}(t), \tilde{r}(t), \dot{\tilde{r}}(t)\right) \\ &= F\left(\tilde{r}(t), \dot{\tilde{r}}(t)\right) + K_p\left(\tilde{r}(t) - \tilde{q}(t)\right) + K_d\left(\dot{\tilde{r}}(t) - \dot{\tilde{q}}(t)\right). \end{aligned}$$




## Conclusion

Locally, a mechanical system undergoing impacts is equivalent to a switched system. For perfectly elastic impacts, we develop a local reference tracking controller in the switched system $\tilde{C}$ and show tracking in $\tilde{C}$ is equivalent to tracking in $C$ away from impacts. While the the *mirror law* [4] provides global results, our method provides a potential way to extend to inelastic collisions.

Future work will be extending the control law for cases with inelastic and perfectly plastic impacts.

## References


[1] D. Pekarek, V. Seghete, and T. D. Murphey, "The Projected Hamilton's Principle: Modeling Nonsmooth Mechanics as Switched Systems." unpublished.
[2] M. Saunders, "Geometrical Mechanics v1." Lecture Notes, 1968.
[3] F. Bullo and R. M. Murray, "Tracking for Fully Actuated Mechanical Systems: A Geometric Framework," Automatica, Jan. 1999.
[4] F. Forni, A. Teel, and L. Zaccarian. "Follow the Bouncing Ball: Global Results on Tracking and State Estimation With Impacts," IEEE TAC, June 2013.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