A DATA ASSIMILATION FRAMEWORK FOR 2D IONOSPHERIC ESTIMATION

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Abstract

Increasing availability of large amounts of data and growing sophistication of current numerical models are allowing us to address new challenging problems of the complex space environment. For instance, currently it is quite common to have multiple instruments observing overlapping regions of the ionosphere at the same time, providing potentially useful information for field of unknowns retrieval. Nevertheless, techniques developed in the data assimilation community have not yet been fully considered for static ionospheric reconstruction or estimation problems. Moreover, questions regarding the usefulness of different ionosphere profilers under the data assimilation context are still mostly unexplored, even though addressing them can help significantly with the ill-posedness of the estimations. In this work, several ionosphere profilers are used, based on Chapman and Epstein functions. Their performance to model vertical density profiles are evaluated using historical data from the Jicamarca Radio Observatory. The measurement models we use are total electron content from GPS and LEO satellites and ionosonde sounders. Both Tikhonov and Total Variation regularization are used to generate the reconstructed image.

Theory

TEC is the number of free electrons along the path between two points (satellite – receiver). Mathematically, it can be represented as:

$$TEC = \int_p n_e(s)ds$$

$n_e(s)$ is the electron density; p is the path between the satellite and the receiver.

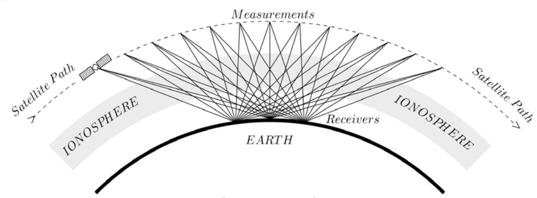
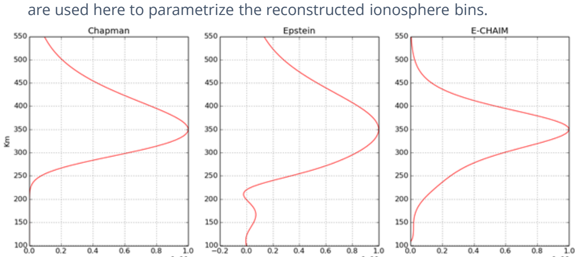


Diagram of a GPS ground receiver system.

Well known analytical ionospheric profilers (Chapman, Epstein, and E-CHAIM) are used here to parametrize the reconstructed ionosphere bins.



Profiles of 3 different ionosphere models: electron density as a function of altitude (km).

Data Assimilation

Chapman

$$N_e(h) = N_m \exp\left[\kappa\left(1 - \frac{h-h_m}{H} - \exp\left(-\frac{h-h_m}{H}\right)\right)\right]$$

E-CHAIM

$$N_e = N_m F_2 \mathrm{sech}^2\left(\frac{h-h_m F_2}{H(h)}\right)$$

Semi-Epstein

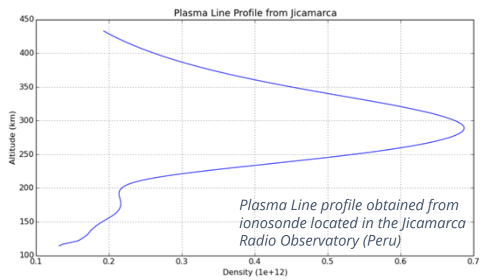
$$N_{F_2}(h) = 4 N_m F_2 \mathrm{sech}^2\left(\frac{h-h_m F_2}{2H_{F_2}}\right)$$

$$H(h) = \frac{H'(h)}{1+\exp\left(\frac{h_m E - 15 - h}{2.5}\right)}$$

$$H'(h) = H_{F_2} + H_{F_1}\mathrm{sech}^2\left(2\frac{h-h_m F_1}{h_m F_2 - h_m F_1}\right) + H_E \mathrm{sech}^2\left(\frac{h-h_m E}{30}\right)$$

Profiles from a quiet Ionosphere can be parametrized as a mix of Chapman, semi-Epstein and E-CHAIM layers.

Ionosonde data provides the peak $F2$ $(N_m F_2)$ and height $(h_m F_2)$ which can be estimated from:



$$N_m F_2 = \frac{f0F2}{8.98^2}$$

$$h_m F_2 = \frac{1490}{M(3000)F2} - 176$$

- $f0F2$ is the critical frequency of the $F2$ layer
- $M(3000)F2$ is the ratio of the maximum usable frequency at a distance of 3,000 km to the $F2$ layer critical frequency, $f0F2$

Reconstruction

Tikhonov

$$\mathbf{J}(\mathbf{x}) = ||\mathbf{y} - \mathbf{H}\mathbf{x}||_2^2 + \alpha_1||\mathbf{D}_1\mathbf{x}||_2^2 + \alpha_2||\mathbf{D}_2(\mathbf{x}-\mathbf{x}_0)||_2^2$$

$$\hat{\mathbf{x}} = (\mathbf{H}^T\mathbf{H} + \alpha_1\mathbf{D}_1^T\mathbf{D}_1 + \alpha_2\mathbf{D}_2^T\mathbf{D}_2)^{-1}(\mathbf{H}^T\mathbf{y} + \alpha_2\mathbf{D}_2^T\mathbf{D}_2\mathbf{x}_0)$$

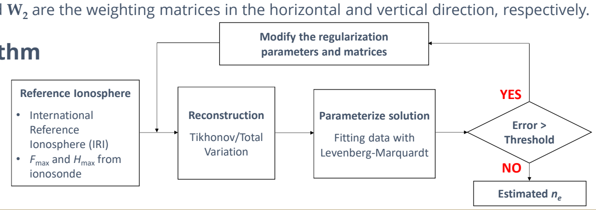
Total Variation

$$\mathbf{J}(\mathbf{x}) = ||\mathbf{y} - \mathbf{H}\mathbf{x}||_2^2 + \alpha_1||\mathbf{D}_1\mathbf{x}||_1 + \alpha_2||\mathbf{D}_2(\mathbf{x}-\mathbf{x}_0)||_1$$

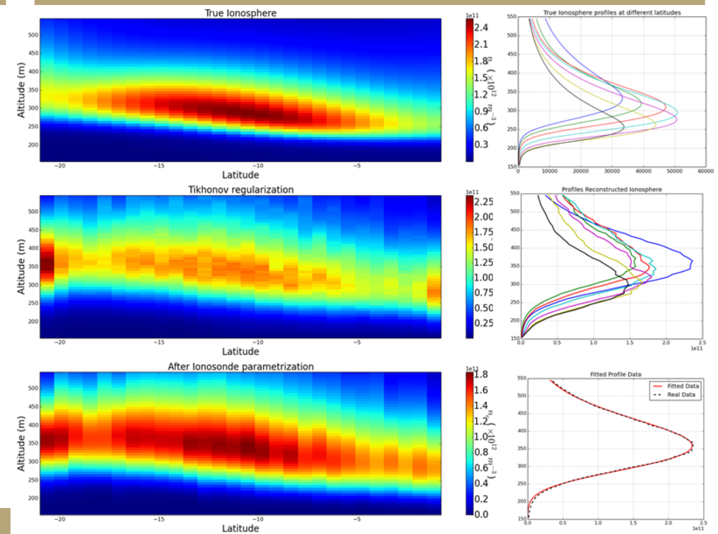
$$\mathbf{H}^T\mathbf{H} + \frac{\alpha_1}{2}\mathbf{D}_1^T\mathbf{W}_1(\hat{\mathbf{x}})\mathbf{D}_1 + \frac{\alpha_2}{2}\mathbf{D}_2^T\mathbf{W}_2(\mathbf{x},\hat{\mathbf{x}}_0)\mathbf{D}_2\hat{\mathbf{x}} = \mathbf{H}^T\mathbf{y} + \frac{\alpha_2}{2}\mathbf{D}_2^T\mathbf{W}_2(\mathbf{x},\hat{\mathbf{x}}_0)\mathbf{D}_2\mathbf{x}_0$$

- α_1 and α_2 are the regularization parameters
- $\mathbf{D}_1$ and $\mathbf{D}_2$ are the regularization matrices
- $\mathbf{x}_0$ is the reference image
- $\mathbf{W}_1$ and $\mathbf{W}_2$ are the weighting matrices in the horizontal and vertical direction, respectively.

Algorithm



Results and Discussion



(top) True ionosphere to reconstruct, (middle) Reconstructed ionosphere using a regular Tikhonov process, (bottom) Reconstructed Ionosphere using the ionosonde reference and fitted to a mix of Chapman-Epstein-E CHAIM model.

Observations

- Combining complementary measurements by ionosonde and GPS-TEC receivers from the perspective of data assimilation improves the quality of the 2D Electron Density image.
- While the TEC measurement represents a tomographic projection, the ionosonde measurement provides values of both the maximum electron density and the maximum height of the F2 layer, which define the priori information for the reconstruction process.
- The regularization method with distinct vertical and horizontal penalization and fitting the data to a mix of ionospheric profile models permits an adequate reconstruction and preserves the ionospheric structure.

Future Work and References

- Reconstruct more challenging ionospheres, especially ones that show up during the night at low latitudes.
- Use data from different instruments (ultraviolet Imagers, incoherent and coherent radars) in order to use more sophisticated data fusion algorithms.

1. Sydney Chapman "The absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth." *Proceedings of the Physical Society* (1926-1948), 43.1, (1931), p. 26.
2. Semeter, Joshua, et al. "GNSS-ISR data fusion: General framework with application to the high-latitude ionosphere." *Radio Science* 51.3 (2016), pp. 118-129.
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