Analyzing communication and flow processes in the brain has received much attention in past years and better models may be crucial for developing treatments for neurological diseases.

Graph signal processing (GSP) has been proposed as a way to study neural signals observed at the nodes of an underlying brain graph [1]. Instead of focusing on node signals, we propose a new framework that models neural communication as flow signals defined at the edges of a graph.

This new perspective allows us to model neural communication flow on a finer temporal scale than other techniques such as Granger causality that are used in neuroscience to date.

**Estimating Neural Flow from LFP Time Series**

**Goal:**
- Estimate neural flow along the edges of a graph (Fig. 1 – right) from local field potential (LFP) time series observed at the graph nodes (Fig. 1 – left).

**Method:**
1. Assume a graph with \( N \) nodes and \( E \) directed edges. Furthermore, time series of neural activity are measured at the nodes of the graph (Fig. 1 – left).
2. Model neural flow through a parameterized diffusion process (Fig. 1 – middle).

\[
\frac{\partial v^i}{\partial t} = \nabla^T \mathbf{W}^i \nabla v^i + \mathbf{B}(v^i) \mathbf{w}^i \quad \forall i \in [N]
\]

- \( \mathbf{w}^i \) is the estimated flow for node \( i \). \( \mathbf{W}^i \) is the weighted Laplacian for node \( i \).
- \( \mathbf{B}(v^i) \) is a function that captures the spatial diffusion of flow across edges and is estimated using a supervised learning approach.

**Fig. 1:** Example of diffusion model. The graph shows from left to right the node signal, the model parameter, and the edge flow respectively. The equation at the bottom illustrates the diffusion model for Node 4.

**Physical interpretation of flow term**
- \( \mathbf{B}(v^i) \mathbf{w}^i \): computes the voltage gradient for each edge
- \( \mathbf{B}(v^i) \mathbf{w}^i \mathbf{x}^i \): computes the edge flow, \( \mathbf{w}^i \) can be interpreted as the conductivity between two nodes, so that conductivity times potential gradient yield the current
- \( \mathbf{B}(v^i) \mathbf{w}^i \mathbf{x}^i \mathbf{W}^i \mathbf{x}^i \): computes the net flow, i.e., the sum of all inflows minus sum of all outflows, for each node

**Motivation**

- The neural flow is estimated from LFP measurements of neural activity of two rhesus macaque monkeys during a stimulation experiment [2].
- Location of stimulation varies between 63 experimental sessions.
- LFPs are measured by 96 electrode micro-electrodecorticography (µ-ECoG) array.
- The ECoG array is used to construct sparsely connected graph, where each node is connected to approximately its 8 nearest neighbors.

**Validation: Comparing Flow with No-flow Baseline Model**

- No-flow baseline model: same as flow model but with constraint \( w^i = 0 \)
- Flow and no-flow model fitted to all 63 sessions to estimate \( w^i \) and \( \mathbf{W}^i \)
- Estimated model parameter used for one-step-ahead prediction on test dataset not included during model fitting.
- Compute root-mean square error (RMSE) between measured LFP \( z(t) \) and predicted LFP \( \hat{z}(t) \)
- Relative improvement of flow over no-flow model for each session \( i \):

\[
I(i) = \frac{\text{RMSE}_{	ext{flow}} - \text{RMSE}_{	ext{no flow}}}{\text{RMSE}_{	ext{no flow}}} \times 100
\]

**Fig. 2:** Normalized neural flow of a single experimental session 5 and 15 ms after stimulation. The location of stimulation is indicated by the black cross.

**Future Work and References**

- Build on these results and demonstrate how GSP can be further adapted to analyze flow signals.
- Investigate the relation between our estimated flow and other brain connectivity measures such as Granger causality.

**REFERENCES**

