CONVEX CONTROL OF GRID-INTERFACING INVERTERS WITH CURRENT LIMITS

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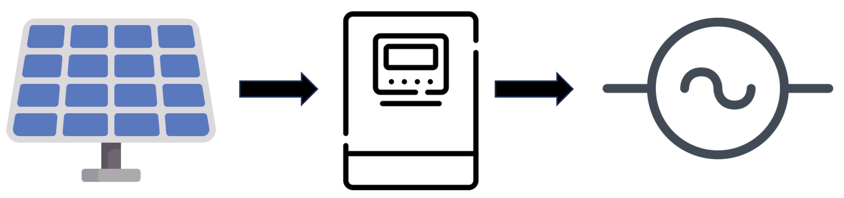
CHALLENGES OF INVERTER CONTROL

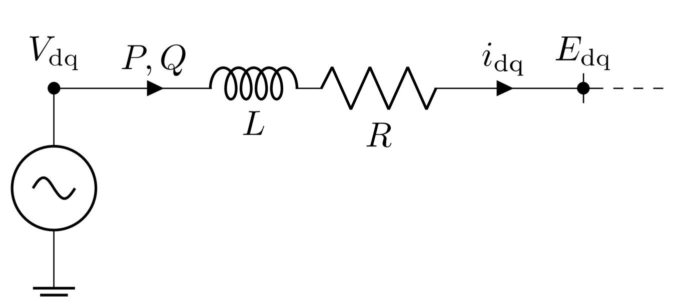
- Inverters connect renewables such as solar and storage to the grid, so they are quickly becoming an integral part of the power system.
- Unlike traditional generators, inverter-based resources are smaller and tend to saturate much more easily.



CONTRIBUTIONS

- Existing control methods mostly rely on PI controllers, which are easy to implement but cannot explicitly handle hard constraints, for example, limits on current magnitudes.
- They are either designed to be conservative or result in constraint violations.
- The optimal control problem was thought to be difficult because of the nonlinear relationships between voltage, current and power, but we show that there are surprising convexities relating these quantities, allowing us to design efficient and safe controllers.

MODEL FORMULATION



We model the inverter as a voltage source $\boldsymbol{V}_{\mathbf{dq}}$ connected through inductor L and resistor R to an infinite bus with constant voltage $\boldsymbol{E}_{\mathbf{dq}}^T = [E \;\; 0]$ and frequency ω. The current dynamics in the dq-reference frame of the grid are governed by:

$$\begin{bmatrix} \dot{i}_{\mathrm{d}} \\ \dot{i}_{\mathrm{q}} \end{bmatrix} = \begin{bmatrix} -R/L & \omega \\ -\omega & -R/L \end{bmatrix} \begin{bmatrix} i_{\mathrm{d}} \\ i_{\mathrm{q}} \end{bmatrix} + \frac{1}{L} \begin{bmatrix} V_{\mathrm{d}} - E \\ V_{\mathrm{q}} \end{bmatrix}$$

At equilibrium, $$\begin{bmatrix} \dot{i}_{\mathrm{d}} \\ \dot{i}_{\mathrm{q}} \end{bmatrix} = \mathbf{0} \Rightarrow \boldsymbol{V}_{\mathbf{dq}}^{\boldsymbol{eq}} = \boldsymbol{E}_{\mathbf{dq}} - L \begin{bmatrix} -R/L & \omega \\ -\omega & -R/L \end{bmatrix} \boldsymbol{i}_{\mathbf{dq}}^{\boldsymbol{eq}}.$$

We calculate the inverter's active (*P*) and reactive (*Q*) power as:

$$P = \frac{3}{2}\, \boldsymbol{i}_{\mathbf{dq}}^T \boldsymbol{V}_{\mathbf{dq}}\,, \qquad Q = \frac{3}{2}\, \boldsymbol{i}_{\mathbf{dq}}^T \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \boldsymbol{V}_{\mathbf{dq}}$$

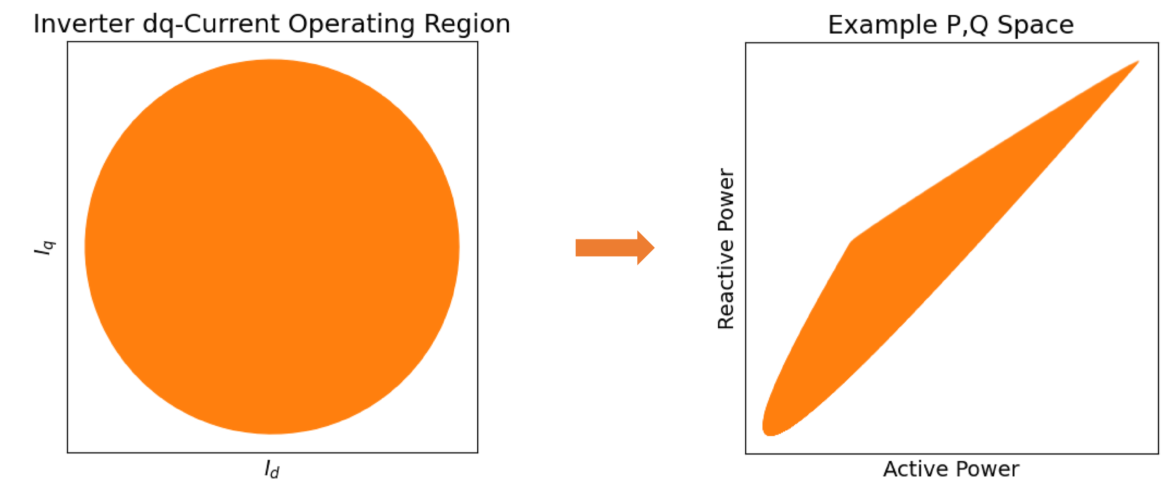
The goal of our controller is to track a given power/voltage setpoint as closely as possible without exceeding inverter current limits or degrading system stability.

CONVEXITY OF THE FEASIBLE OPERATING REGION

Theorem 1. The set $\mathcal{C} = \{(\alpha \boldsymbol{x}^T\boldsymbol{x} + \boldsymbol{a}^T\boldsymbol{x}, \beta \boldsymbol{x}^T\boldsymbol{x} + \boldsymbol{b}^T\boldsymbol{x}) | \boldsymbol{x}^T\boldsymbol{x} \le \gamma\}$ is convex $\forall \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{x} \in \mathbb{R}^2, \forall \alpha, \beta, \gamma \in \mathbb{R}_+$.

Each of an inverter's operating regions defined by $(P, Q), (P, V_{\mathrm{dq}}^2)$, and (Q, V_{dq}^2) can be written in the form of set $\mathcal{C}$ with $\boldsymbol{x}^T = [i_{\mathrm{d}} \;\; i_{\mathrm{q}}]$. Thus, Theorem 1 guarantees convexity, enabling the implementation of our optimal control algorithm.

Shown below are corresponding sets $\{\boldsymbol{i}_{\mathbf{dq}} \in \mathbb{R}^2 | \|\boldsymbol{i}_{\mathbf{dq}}\|_2^2 \le I_{nom}\}$ (*left*) and $\{(P, Q) | \|\boldsymbol{i}_{\mathbf{dq}}\|_2^2 \le I_{nom}\}$ (*right*) for random vectors $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^2$ used to generate (P, Q).



OPTIMAL CONTROLLER ALGORITHM

Following a disturbance, or change to the setpoint (P_{ref}, Q_{ref}), we determine the optimal inverter setpoint according to

$$\min_{\|\boldsymbol{i}_{\mathbf{dq}}\|_2 \le 1} \;\; \|P - P_{ref}\|_2^2 + \lambda \|Q - Q_{ref}\|_2^2\,.$$

where $\|\boldsymbol{i}_{\boldsymbol{dq}}\|_2 \le 1$ represents the current magnitude limits (normalized to 1) and λ is a customizable weighting between the two setpoints to handle situations when one or both setpoints cannot be reached simultaneously. The same problem can be adapted to track $(P_{ref}, V_{\mathrm{dq},ref}^2)$ or $(Q_{ref}, V_{\mathrm{dq},ref}^2)$ setpoints instead.

By convexity, we know this is equivalent to solving the Semidefinite Program:

$$\min_{\boldsymbol{W} \succcurlyeq 0} \; \|Tr(\boldsymbol{AW}) - P_{ref}\|_2^2 + \alpha \|Tr(\boldsymbol{BW}) - Q_{ref}\|_2^2 \; s.t. \; W_{11} + W_{22} \le 1, W_{33} = 0$$

and recovering $\boldsymbol{i}_{\mathbf{dq}}^*$ from $\boldsymbol{W}^*$, allowing the problem to be solved efficiently.

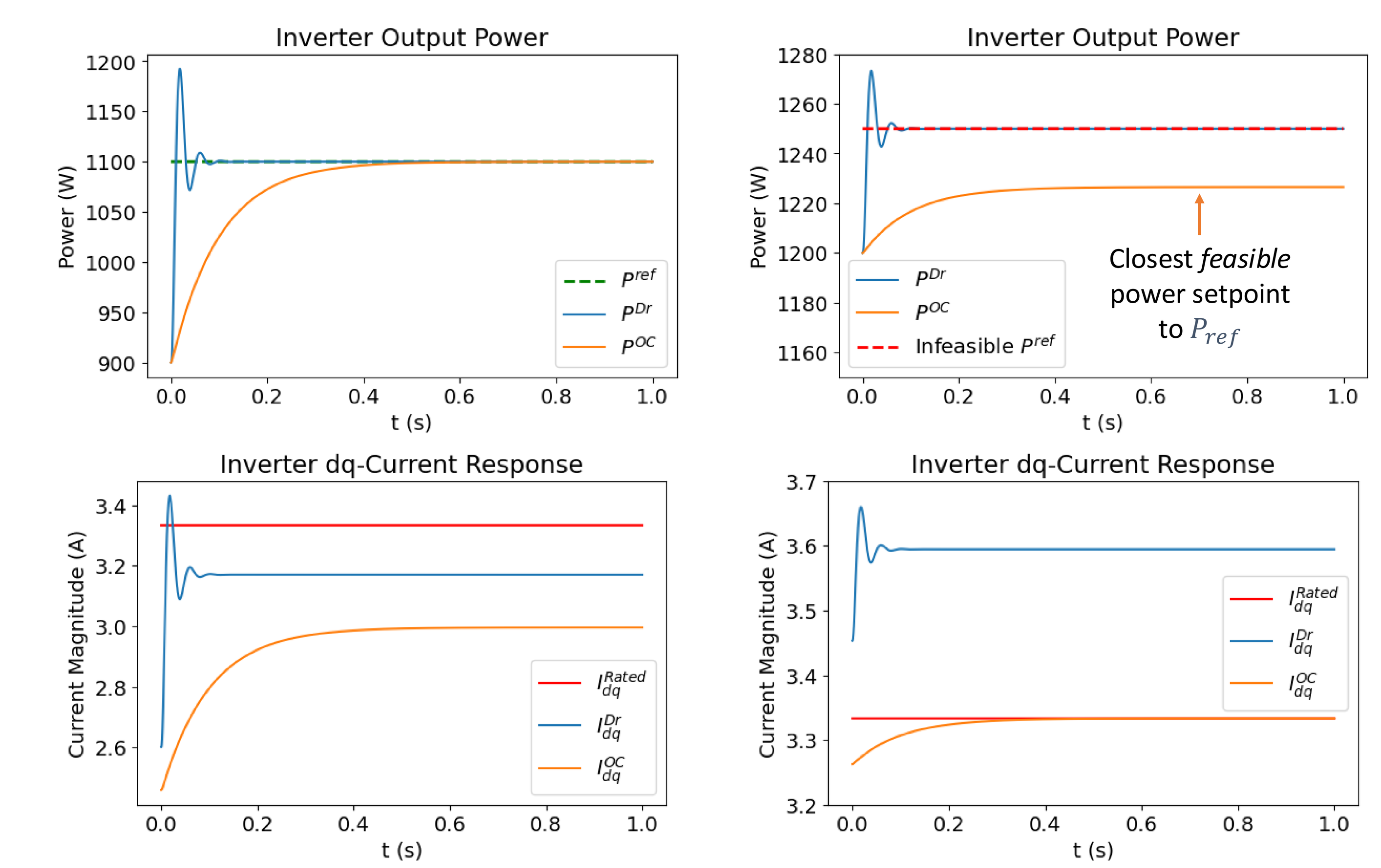
If (P_{ref}, Q_{ref}) lies outside the inverter's operating region, the optimal solution is found by projecting (P_{ref}, Q_{ref}) onto the set of feasible $\{(P, Q) | \|\boldsymbol{i}_{\mathbf{dq}}\|_2 \le 1\}$ and determining the corresponding $\boldsymbol{i}_{\mathbf{dq}}^*$.

Once we have our new $\boldsymbol{i}_{\mathbf{dq}}^*$, we determine the corresponding $\boldsymbol{V}_{\mathbf{dq}}^*$ at equilibrium and implement a linear feedback controller on the inverter voltage to reach $\boldsymbol{V}_{\mathbf{dq}}^*$.

CASE STUDY RESULTS

We compared the responses of our optimal controller (**orange**) to a standard droop controller (**blue**) following two step-changes in the active power setpoint: one that remains within the feasible operating region and one that exceeds the inverter's ratings.

Feasible P_{ref} Setpoint — Infeasible P_{ref} Setpoint



These results demonstrate the improved performance over droop control and highlight the following features of our optimal control strategy:

- Ensures transient currents remain within the safe operating region of the inverter.
- Maximizes inverter power and current without exceeding current ratings when tracking an infeasible power setpoint.
- Chooses the optimal inverter voltage to achieve desired power output while reducing current, and thus losses.

Future Work, References, and Acknowledgments

Future work:

- Implement a dynamic optimization process to simultaneously solve for and approach the optimal setpoint.
- Investigate the convexity of feasible operating regions in larger, networked systems of inverters.
- Demonstrate controller implementation and performance on a testbed system.

References:

[1] T. Joswig-Jones and B. Zhang, "Optimal control of grid-interfacing inverters with current magnitude limits," in *2024 IEEE 63rd Conference on Decision and Control (CDC)*, 2024, pp. 4623–4628.

[2] T. Joswig-Jones and B. Zhang, "Safe control of grid-interfacing inverters with current magnitude limits," 2024. [Online]. Available: https://arxiv.org/abs/2409.13890

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