HOW MUCH TIMING ERROR CAN YOU TOLERATE BEFORE TRUST BREAKS?

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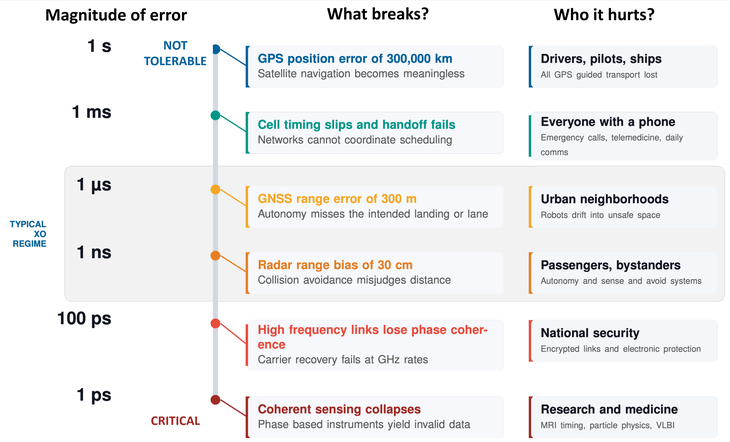
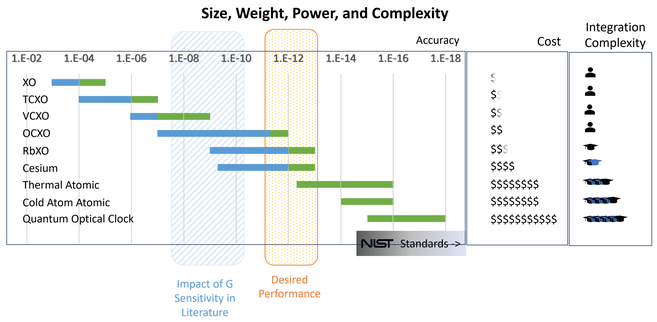
AUTONOMY, ROBOTICS, & CONNECTIVITY

WHEN YOUR CLOCK DRIFTS



UAV navigation often needs sub-microsecond synchronization
A low-cost oscillator can drift beyond 10x that under modest perturbation

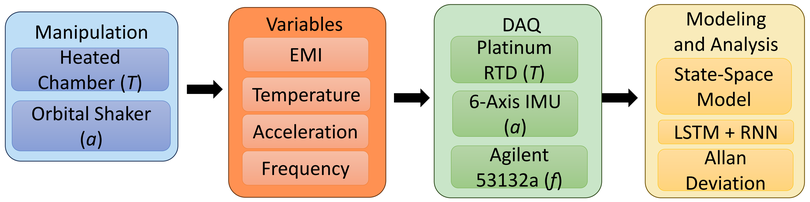
Why not use a Better clock?



Alternative timekeeping methods are also unsuitable, expensive and heavy

Our Goal and Design

The goal: using sensor fusion, give a $10 oscillator the stability and resilience of a $1,000+ oscillator.



Methodology

$$p(t) = \begin{bmatrix} T(t) \\ V(t) \\ C_L(t) \\ I(t) \\ P(t) \\ H(t) \\ a_x(t) \\ a_y(t) \\ a_z(t) \end{bmatrix}$$

Temperature
Supply Voltage
Load capcitance
Control current
Pressure
Humidity
Tri − axial acceleration component

Non-linear Responses → Higher Order Polynomials
Coupled Environmental Channels → Cross Terms
Delayed Impacts → Lag States

Gap in Literature

Mathematical Strategy

$$\Delta p(t) \rightarrow \boldsymbol{\varphi(t)} = \begin{bmatrix} 1 \\ \text{Linear Terms} \\ \text{Quadratic} \\ \vdots \\ \text{Cross} - \text{terms} \end{bmatrix}$$

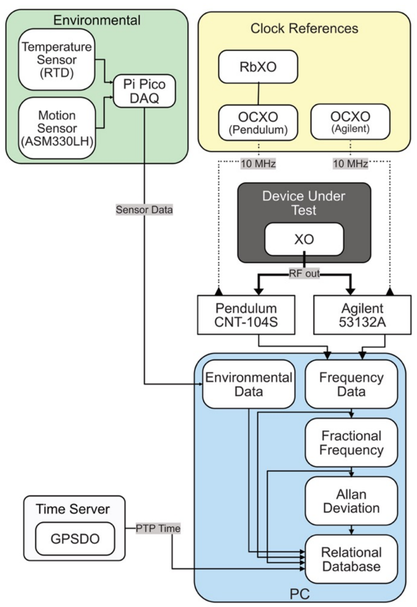
Lifted Feature Vector

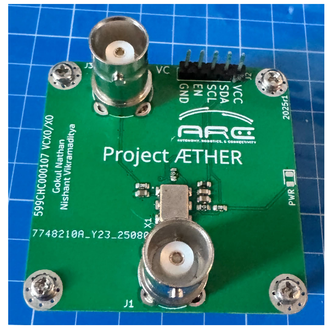
$$\dot{\boldsymbol{\theta}}(t) = \boldsymbol{A_\theta}\,\boldsymbol{\theta}(t) + \boldsymbol{B_\theta}\,\boldsymbol{\varphi}(t) + w_\theta(t)$$

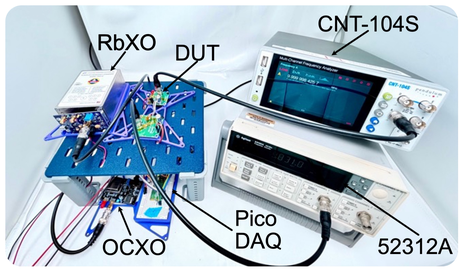
Drift · *Sentivity Matrix* · *Environmental Forcing* · Noise

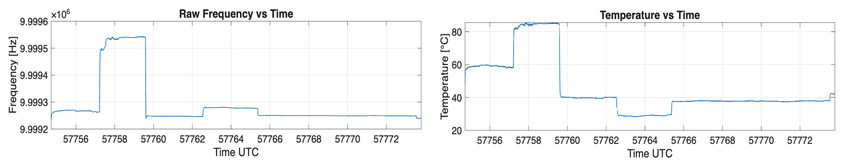
$$\gamma(t) = \boldsymbol{\gamma}^\top \boldsymbol{\theta}(t) + \boldsymbol{D_y}\,\boldsymbol{\varphi}(t) + \epsilon_y(t)$$

Fractional *Frequency*



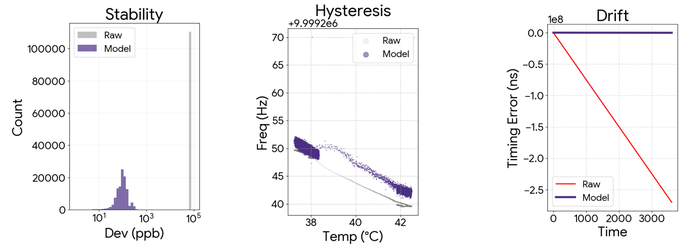






Hardware and Data Correlation Assessment and Validation

Our Insight and Model Capability



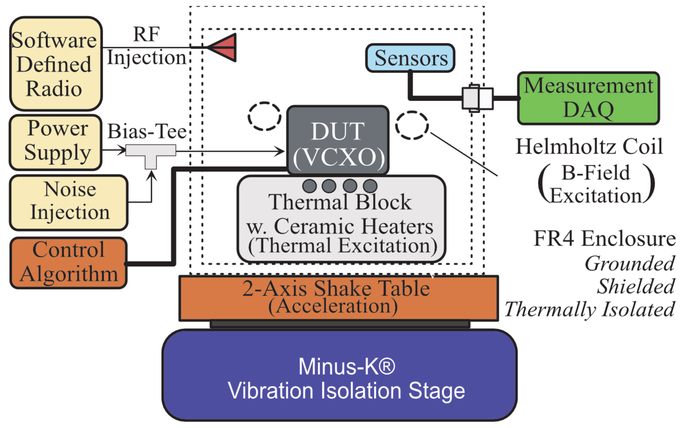
$$A = \begin{bmatrix} 0.9608 & 1.323 & 0.032 \\ 1.016 & 1.2203 & 0.8174 \\ 0 & 0.9814 & 0.9996 \end{bmatrix} \qquad C = \begin{bmatrix} -88.7537 & -9.9616 & 2.4081 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.392 & 0.003 \\ 0.040 & 0 \\ 0.04 & 0.034 \end{bmatrix} \qquad D_y = \begin{bmatrix} 235.31 \\ -53.37 \\ 17.11 \\ 1.87 \\ 0.14 \\ -24.13 \\ -5.93 \end{bmatrix} \begin{matrix} T^3 \\ T^2 \\ T \\ a \\ \dot{\omega} \\ T{\times}a_z \\ T{\times}a_{xy} \end{matrix}$$

Metric	Uncompensated (Raw)		Model-Compensated	Improvement Observed
Fractional Stability	$3.2\text{x}10^{-5}$	→	$9.60\text{x}10^{-8}$	**325x better**
Frequency Deviation	31181 ppb	→	96.02 ppb	**99% reduction**
Timing Drift	31.2 µs	→	96 ns	**31000 ns saved**

Statistical Metric	Value	Control Metric	Status and Implications
RMSE	**1.211 Hz @10Mhz**	Controllability	**Full Rank**
MAPE	**$1.067\text{x}10^{-7}$**	Observability	**Full Rank**
95% PI coverage	**99.89%**	System State	**Fully Identified**
AIC-BIC	**42303-42457**	Forecasting	**Stable for out of state sample**

Future Work



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