

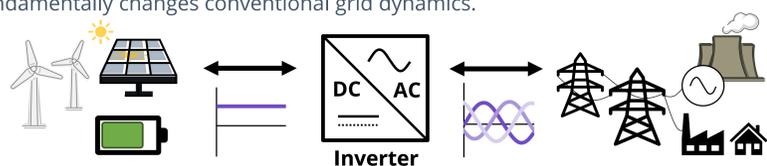


System Strength Sensitivity to Power Flow Perturbations in AC Power Systems with Inverter-based Resources

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Grid-Interfacing Inverters

- **Inverter-based resources (IBRs)**, such as solar, wind, and batteries, produce direct current (DC) and are interfaced to alternating current (AC) networks through inverters.
- Unlike synchronous generators (SGs), inverters lack physical inertia which fundamentally changes conventional grid dynamics.



Power System Strength

System Strength is the ability of power systems to maintain a stable voltage and frequency under disturbances [1].

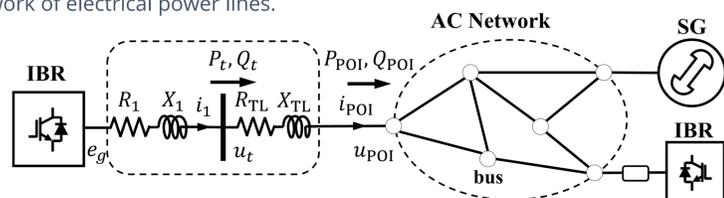
Challenges with System Strength:

- As more IBRs have been integrated into the grid, system strength has decreased.
- Conventional metrics, like the Short-Circuit Ratio (SCR), rely on steady-state fault currents and ignore the dynamics of inverter controls.
- Few system strength metrics provide system operators with clear actionable information to help maintain system stability in real-time.

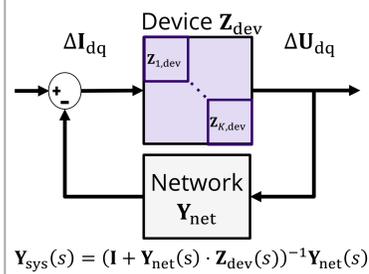
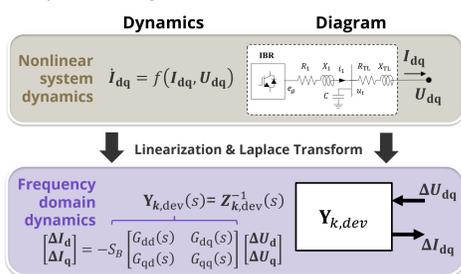
We propose sensitivity indices that trace small-signal stability directly back to specific power injections, highlighting "weak" areas and giving actionable insights.

AC Network System Model

- We model a system with K devices (SGs, IBRs, etc.) at N buses connected through an AC network of electrical power lines.

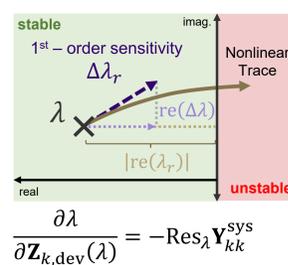


- We model **small-signal system behavior** by linearizing nonlinear device dynamics around a steady-state AC power flow operating point $\mathcal{M} = (P, Q, V, \theta)$, which represent **equivalent impedance models** in the frequency domain.
- Coupling these device admittances with the network admittance model, yields a closed-loop, **whole-system admittance model** [2].



Small-signal Stability Sensitivity Analysis

- This model allows us to evaluate **if the system is stable, returning to its equilibrium after small disturbances.**
- System stability is determined by its poles; **if all poles reside in the left-half of the complex plane, the system is stable.**



- **First-order pole sensitivities** to device impedances can be derived from the residue of the whole system admittance model and used to approximate how susceptible the system small-signal stability is to a perturbation in Z_{dev} [3].

$$\frac{\partial \lambda}{\partial Z_{k,dev}(\lambda)} = -\text{Res}_{\lambda} Y_{kk}^{sys}$$

Power Flow Sensitivities

- **Perturbing the power injection at one bus perturbs other buses** resulting in a new operating point that satisfies the steady-state power flow equations

$$P_i = V_i \sum_{j=1}^N V_j (G_{kj} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j))$$

$$Q_i = V_i \sum_{j=1}^N V_j (G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j))$$

Power Flow Jacobian Partition

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta V \\ \Delta \theta \end{bmatrix}, \quad J = \begin{bmatrix} \frac{\partial P}{\partial V} & \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} & \frac{\partial P}{\partial \theta} \\ \frac{\partial Q}{\partial V} & \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} & \frac{\partial Q}{\partial \theta} \end{bmatrix} = \begin{bmatrix} J_{V-} & J_{\theta-} \\ J_{V-} & J_{\theta-} \end{bmatrix}$$

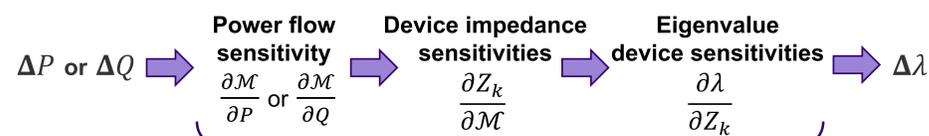
System Power Flow Perturbation Calculation

$$\begin{bmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial \theta}{\partial x} \end{bmatrix} = J^{-1} \begin{bmatrix} \frac{\partial P}{\partial x} \\ \frac{\partial Q}{\partial x} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\partial P}{\partial x} \\ \frac{\partial Q}{\partial x} \end{bmatrix} = \begin{bmatrix} J_{V-} & J_{\theta-} \\ J_{V-} & J_{\theta-} \end{bmatrix} \begin{bmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial \theta}{\partial x} \end{bmatrix}$$

$$x = \bar{P}_k \text{ or } \bar{Q}_k \rightarrow \frac{\partial \mathcal{M}}{\partial x} = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial Q}{\partial x} & \frac{\partial V}{\partial x} & \frac{\partial \theta}{\partial x} \end{pmatrix}$$

Power Flow → Eigenvalue Sensitivity Indices

- **Changes in the power flow result in a change in equivalent device impedances** as we must re-linearize around the new operating point for each device in the system.
- Chaining first-order sensitivities together **we can calculate the sensitivity of the system's poles to a perturbation in the power injection at a bus.**



Apply Chain Rule to obtain Eigenvalue sensitivity to Power Injection

$$\frac{\partial \lambda}{\partial P} \text{ or } \frac{\partial \lambda}{\partial Q}$$

- Using these pole sensitivities to power injections **we define a modal power flow sensitivity index, and metrics relating to the maximum normalized pole shift towards the right-hand plane** for an increase or decrease in a power injection value at bus i as

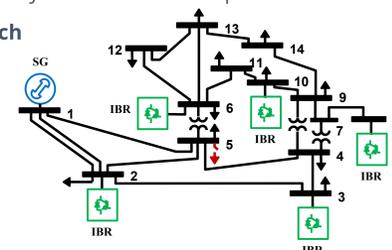
$$\mu SI_{i,\lambda} = \frac{\text{re}(\Delta \lambda^{\mu i})}{|\text{re}(\lambda^{\mu i})|}$$

$$\mu TSI_i = \max_{\lambda} \mu SI_{i,\lambda}, \quad \mu FSI_i = \max_{\lambda} -\mu SI_{i,\lambda}$$

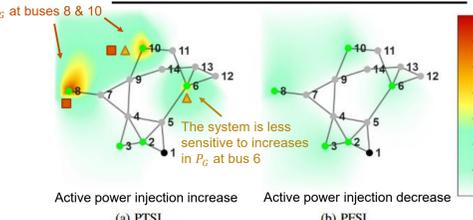
Case Study

- We test our approach with the IEEE 14-bus system with synch. condensers replaced with IBRs.

- Calculating **the power flow sensitivity indices at each bus identifies sensitive/weak buses.**
- We validate our proposed metrics' abilities to identify weak buses by simulating the system's response to a load step at bus 5 for various power injection cases.

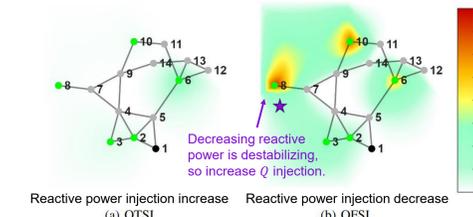


Base Case Active Power Sensitivities

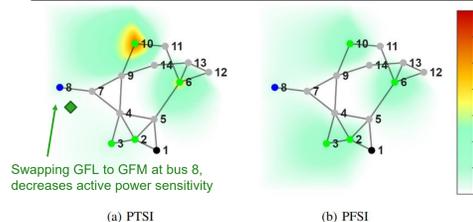


- These sensitivity indices can also be used to **identify remedial actions that system operators can take to improve stability.**

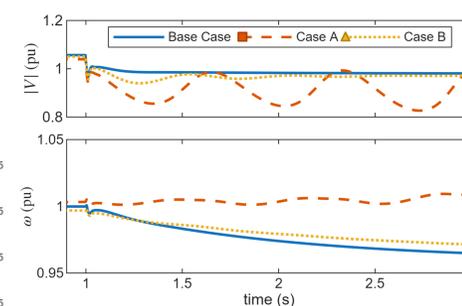
Case A Reactive Power Sensitivities



Base Case Active Power Sensitivities with GFM at Bus 8



Case	Changes from Base Case
Case A	PG : 0.5 pu → 1.2 pu at bus 8 and bus 10
Case B	PG : 0.5 pu → 1.2 pu at bus 6 and bus 10
Case A-2	PG : 0.5 pu → 1.2 pu at bus 8 and bus 10, QG : 0.05 pu → 0.5 pu at bus 8.
Case A-3	PG : 0.5 pu → 1.2 pu at bus 8 and bus 10, Bus 8 GFL to GFM control.



Future Work and References

- We will extend this work to consider sensitivities to changes in line admittance to **predict critical N-1 line outage cases, and dangerous or valuable line switching actions.**

[1] T. Joswig-Jones, S. Dong, J. Tan, D. Wu, and B. Zhang, "A review of system strength metrics for inverter-based power systems," 2026, accepted for publication at PES-CM 2026.
 [2] Y. Gu, Y. Li, Y. Zhu, and T. C. Green, "Impedance-based whole-system modeling for a composite grid via embedding of frame dynamics," IEEE Trans. Power Syst., vol. 36, no. 1, p. 336–345, Jan. 2021.
 [3] Y. Zhu, Y. Gu, Y. Li, and T. C. Green, "Participation analysis in impedance models: The grey-box approach for power system stability," IEEE Trans. Power Syst., vol. 37, no. 1, pp. 343–353, 2022.