Implementation of Extended Stabilizer Simulator for Realistic Noise Modeling

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Introduction

- Quantum computing offers potential speedups but is sensitive to noise.
- Understanding the effects of noise requires efficient simulation of quantum error correcting (QEC) systems.
- Stabilizer simulators represent quantum states by their stabilizer generators instead of explicitly storing all d^n amplitudes. Under Clifford operations (Fourier, Phase, and SUM gates), the stabilizer tableau of size $2n \times (2n+1)$ can be updated efficiently, enabling polynomial-time simulation of large qudit systems.

$$S = \omega^c \bigotimes_{k=1}^{n} X_k^{a_k} Z_k^{b_k}$$

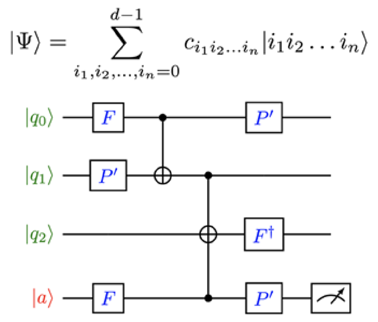
$$\mathcal{S} = \langle S_1, S_2, \ldots, S_m \rangle$$

$$\mathrm{SUM}\,(|i\rangle \otimes |j\rangle) = |i\rangle|(i+j) \bmod d\rangle$$

$$\mathcal{F}|i\rangle = \sum_{j=0}^{d-1} \omega^{ij}|j\rangle$$

$$P'|j\rangle = \omega^{j(j-1)/2}|j\rangle$$

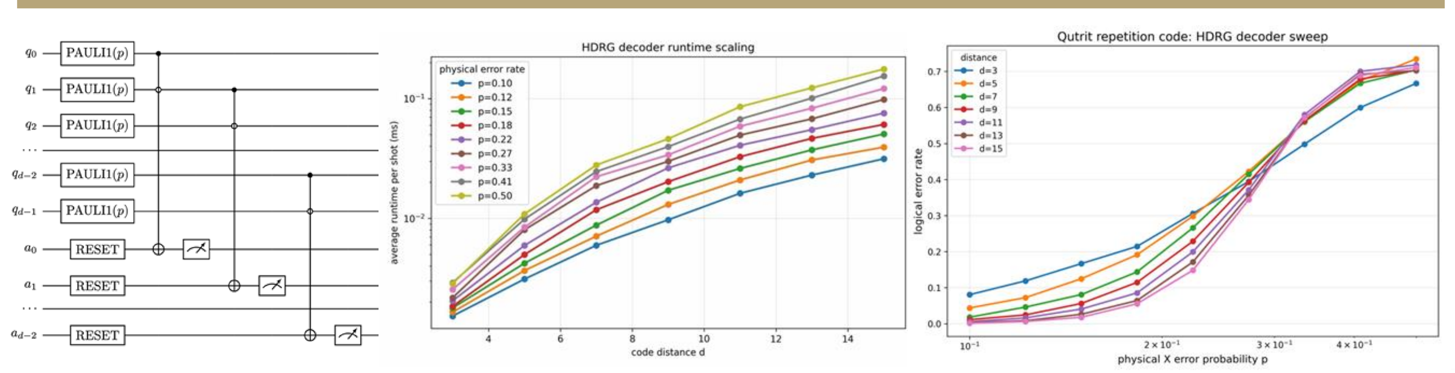
$$|\Psi\rangle = \sum_{i_1,i_2,\ldots,i_n=0}^{d-1} c_{i_1 i_2 \ldots i_n}|i_1 i_2 \ldots i_n\rangle$$



$$\left[\begin{array}{ccc|ccc|c} x_{0,0} & \cdots & x_{0,n-1} & z_{0,0} & \cdots & z_{0,n-1} & r_0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{n-1,0} & \cdots & x_{n-1,n-1} & z_{n-1,0} & \cdots & z_{n-1,n-1} & r_{n-1} \\ \hline x_{n,0} & \cdots & x_{n,n} & z_{n,0} & \cdots & z_{n,n-1} & r_n \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{2n-1,0} & \cdots & x_{2n-1,n-1} & z_{2n-1,0} & \cdots & z_{2n-1,n-1} & r_{2n-1} \end{array}\right]$$

- Most stabilizer simulators built for qubits and qudit simulation can be slow.
  - We implement a high-performance C++ qudit stabilizer simulator and use it to study realistic stabilizer-compatible qudit noise models.
- **Research question:** What is the impact and computational cost of simulating more realistic quantum noise in qudit systems?

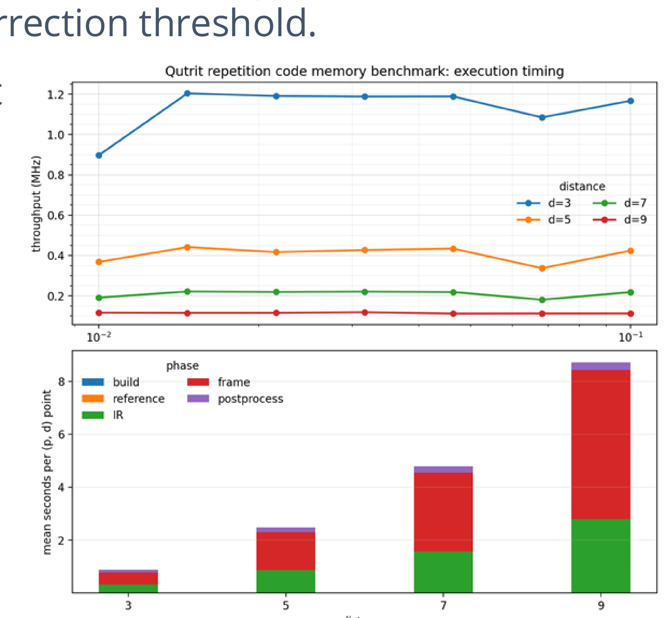
Qutrit Repetition Code and HDRG Decoder



The repetition code was chosen as an initial benchmark for the stabilizer simulator because it uses only bit-flip errors and has a simple, well-understood syndrome structure, enabling validation of gate operations, error insertion, syndrome measurements, and tableau updates before extending to more complex quantum error-correcting codes.

The resulting syndrome patterns are then extracted and decoded using the HDRG decoder. If both the stabilizer simulator and the decoder function correctly, the corrected logical error rate should decrease with increasing code distance when the physical error rate is below the error-correction threshold.

- The simulator maintains nearly constant throughput across the tested error rates, indicating stable performance over the simulation range.
- The HDRG decoder runtime increases gradually with both code distance and physical error rate, demonstrating efficient scaling.
- The logical error rate decreases with increasing code distance below the threshold region, confirming successful error correction by the simulator–decoder pipeline.



Noise model

- **Thermal Relaxation noise channel:**

$$\partial_t\rho = \sum_{n=0}^{d-2}\gamma_n(\langle n_b\rangle + 1)D[|n\rangle\langle n+1|](\rho) + \sum_{n=0}^{d-2}\gamma_n\langle n_b\rangle D[|n+1\rangle\langle n|](\rho) + \sum_{n=0}^{d-1}\gamma_n D[|n\rangle\langle n|](\rho)$$

  - Our exact equation for thermal relaxation noise can't be efficiently modeled with our stabilizer simulator → Need to approximate with Clifford-compatible operators.
- We model thermal noise using two methods: i) the Pauli Twirling Approximation (PTA) and ii) the Quasi Probabilistic Decomposition (QPD).

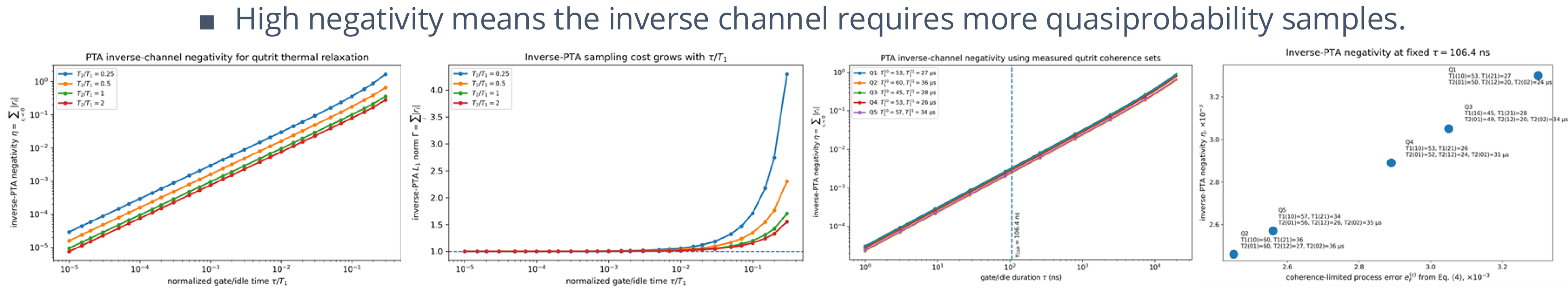
- **Pauli Twirling Approximation (PTA)**
- Exact qutrit thermal relaxation contains non-Clifford amplitude damping and dephasing terms, so it cannot be directly simulated efficiently in a stabilizer framework.
  - PTA approximates the thermal channel by a stochastic qutrit Weyl-error channel:

$$\mathcal{E}_{\mathrm{PTA}}(\rho) = \sum_{a,b=0}^{2} p_{ab} W_{ab}\rho W_{ab}^{\dagger}, \qquad W_{ab} = X^a Z^b$$

  - The PTA channel can be sampled with no negative probabilities. We also analyze the inverse PTA channel, which is useful if PTA were applied to estimate ideal noiseless results from noisy data.
  - To quantify the sampling overhead of the PTA approximation, we compute the inverse channel quasiprobability weights required to represent the inverse of the PTA channel:

$$\eta = \sum_{r_i<0}|r_i|, \qquad \Gamma = \sum_i |r_i|, \qquad \eta = \frac{\Gamma - 1}{2}$$

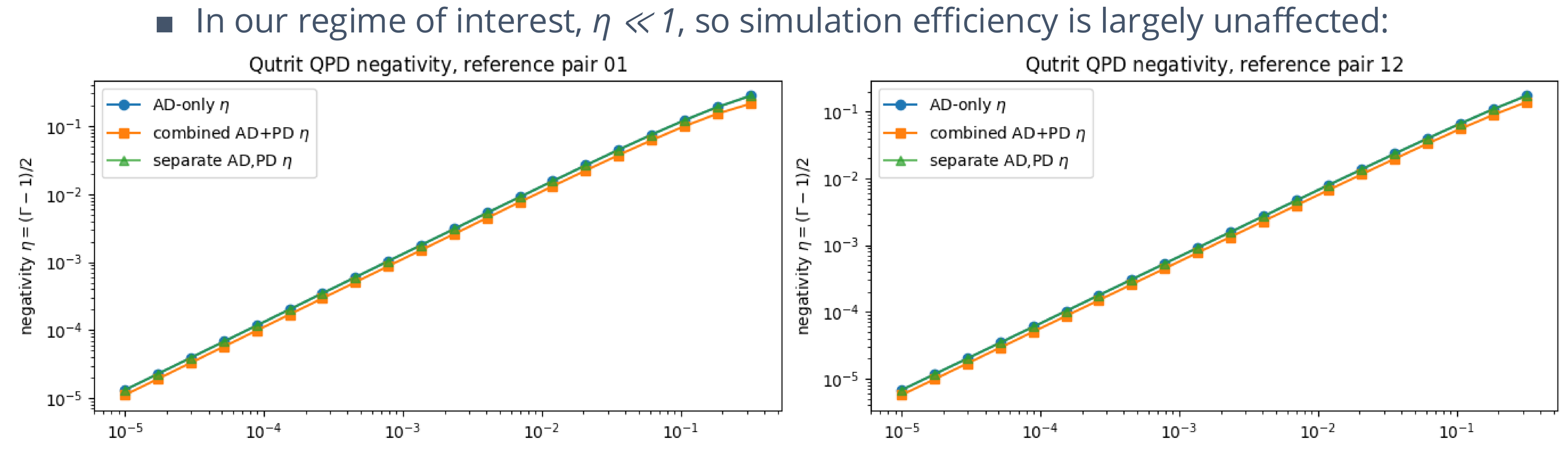
    - High negativity means the inverse channel requires more quasiprobability samples.



- **Quasi Probabilistic Decomposition (QPD)**
  - PTA under/over-estimates noise channel strength depending on T_1 and T_2 times.[1][2]
  - QPD is unbiased because we keep coherent terms in our noise channel.

$$\mathcal{E}_{pt}(\rho) = \sum_i \chi_{i,i}\sigma_i\rho\sigma_i \rightarrow \mathcal{E}_{qpd}(\rho) = \sum_i q_i S(\rho)$$

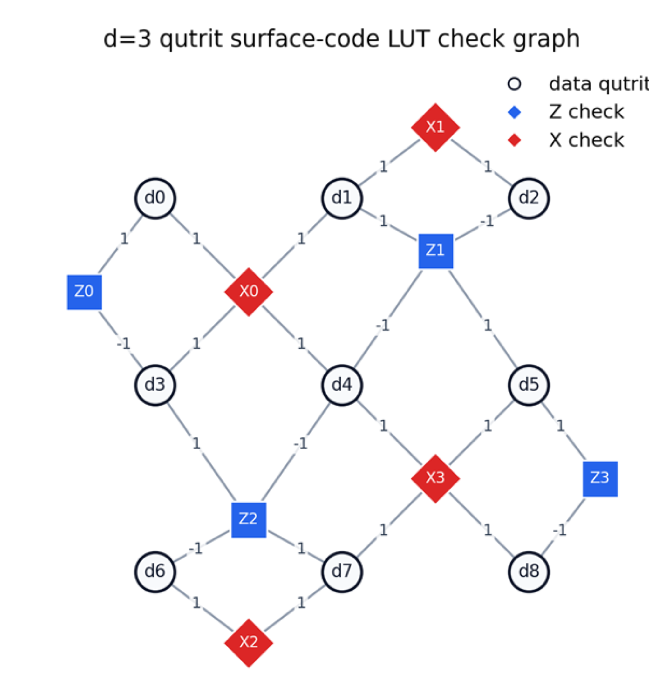
  - Quasi-probabilistic nature of QPD can increase sampling variance, reducing simulation efficiency. Negativity η, a useful figure of merit, quantifies this overhead.
    - In our regime of interest, $\eta \ll 1$, so simulation efficiency is largely unaffected:

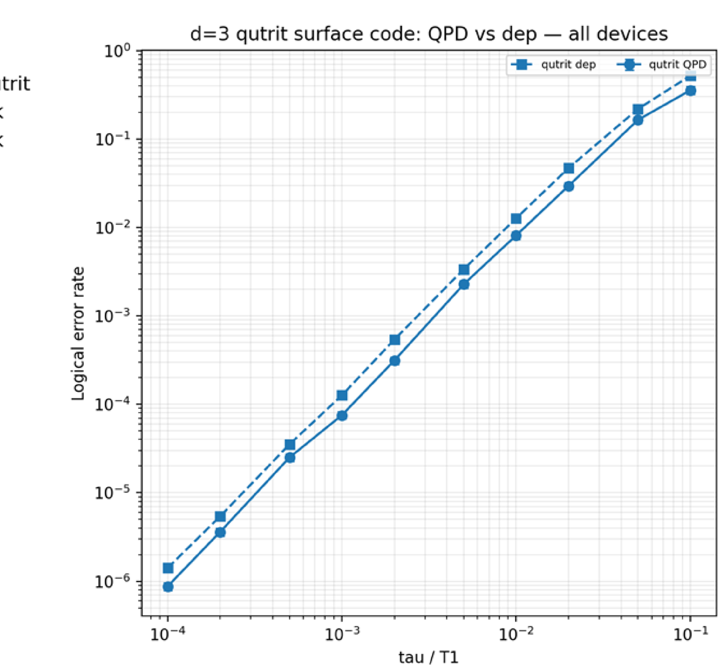


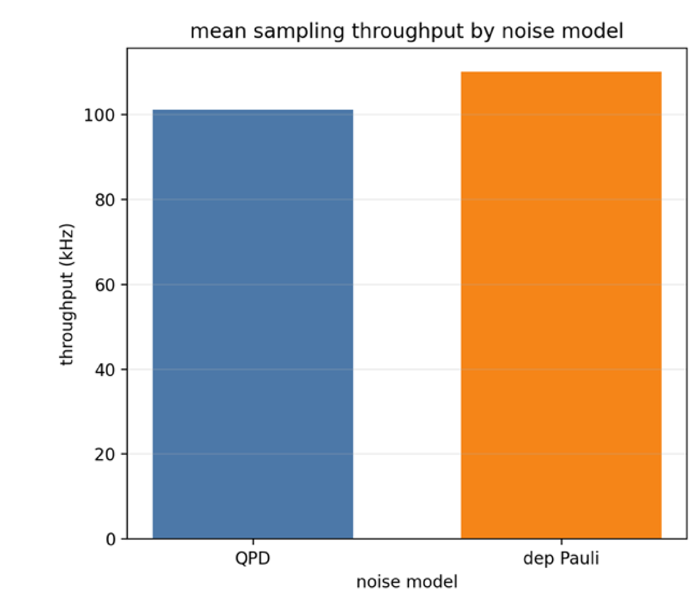
  - However, the QPD extends the operator set from Paulis to Cliffords and reset channels, preventing use of Pauli-frame tracking and increasing simulation cost from $O(m)$ to $O(m^2)$.

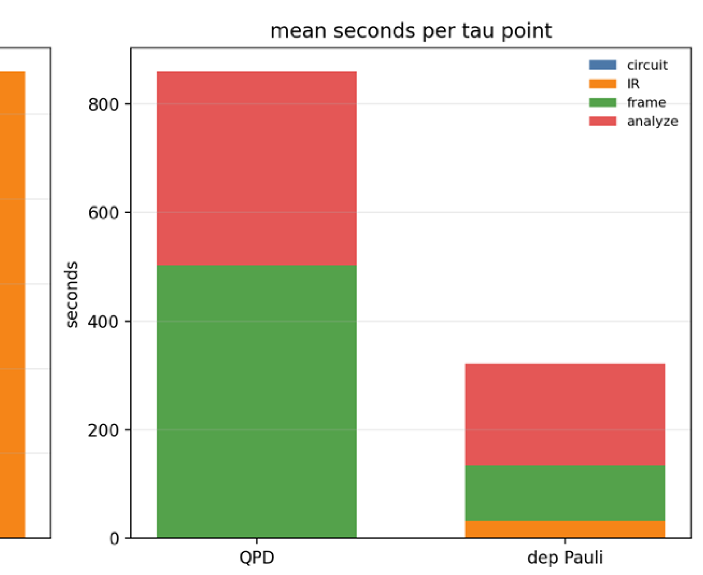
Small Size Qutrit Surface Code and LUT Decoder

- **Goal:** Quantify the impact of rich qutrit noise models on QEC simulations.
  - Benchmark: distance-3 surface-qutrit memory experiment.
    - Computationally tractable while preserving core QEC mechanisms.
  - Noise model: QPD thermal noise parameterized by swept gate duration τ
    - Lookup table decoder; Depolarizing Pauli-noise surrogate baseline.
    - LER computed by reweighting shots with QPD coefficients.









Future Work, References, and Acknowledgments

Future Work

- Compare qutrit QPD results to PTA.
- Use HDRG decoder to analyze larger surface codes and generate threshold plots.

References

[1] Garner, Sean R. and Myers, Nathan M. *et al.* Simulation of thermal-relaxation noise for quantum error correction. *Phys. Rev. Research* **8**, 023134 (2026)
[2] Bravyi, S., Englbrecht, M., König, R. *et al.* Correcting coherent errors with surface codes. *npj Quantum Inf* **4**, 55 (2018)

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