OPTIMAL DESIGN OF EVALUATION POINT IN ERROR-BUDGETING

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Motivation and Abstract

Efficient **quantum error correction (QEC)** is necessary to harness the full potential of quantum computing, and benchmarking the efficiency of QEC experiments is vital to understanding and improving quantum computing.

In QEC, the performance of the decoder, which is responsible for error correction, can be benchmarked by analyzing the **logical error probability (LEP)**, a metric that indicates the frequency of observable correction failures. Estimating the LEP relies on **finding the error scaling parameter, Λ, at different noise parameter values.**

Reducing the variance in the computed values for Λ allows for a more accurate estimation of the LEP. Currently, the issue lies in calculating the gradient of Λ^{-1} at different points, as the distances between the points contributes significantly to variance, so **finding the optimal set of points to evaluate would reduce the number of points needed and improve LEP estimation accuracy.**

We recreate existing results in minimizing Λ^{-1} using the current linear and logarithmic methods of choosing noise parameter values with which to calculate the gradient of the error scaling parameter.

We implement two optimal experimental designs, **D-optimal** and **C-optimal,** to select the noise parameter points with which to calculate the gradient of Λ^{-1}.

The project concludes with **benchmarking the optimal design approaches with the linear and logarithmic point selections**.

Error Thresholds and Scaling

The threshold theorem suggests that the **LEP per shot should exponentially decrease with the size of the code under sufficiently low noise levels.** As such, there exists a physical noise level, or threshold, at which this flips, and increasing the code distance leads to worsening LEP.

Below the threshold, the LEP decreases exponentially with code distance:

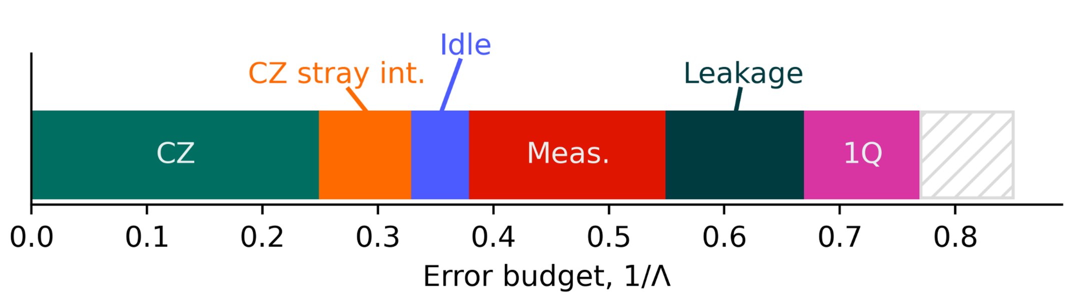
$$\epsilon_d = \frac{1}{\Lambda_0}\Lambda^{-\left\lceil\frac{d+1}{2}\right\rceil}$$

Λ is our error scaling/suppression parameter, and Λ_0 is a constant factor.

Ideally, **Λ should be as high as possible, so we can treat Λ^{-1} as an error "budget"**, that should be as close to 0 as possible.

Riverlane, through their open-source QEC toolkit **Deltakit**, allows us to design, simulate, and benchmark noisy QEC experiments.

Deltakit has implemented an error-budgeting feature that aims to estimate the contribution of individual contributions to the overall error budget. **Reducing the variance of Λ^{-1} will allow for better error budget estimation.**



Optimal Experimental Design

We minimize the variance of the error gradient analysis using **optimal experimental designs**.

The problem we are solving is of the form:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & \vdots & \vdots \\ 1 & x_N & x_N^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$\mathbf{y} = \mathbf{Xa}$$

Where the a_i values are the coefficients of our polynomial fit. The variance of our gradient estimate, $\mathbf{a}$, relates to this problem as:

$$\mathrm{Var}(\mathbf{a}) = \sigma^2 \left(\mathbf{X}^{\mathrm{T}}\mathbf{X}\right)^{-1}$$

Where σ is a constant. Importantly, we see that the variance of $\mathbf{a}$ only depends on $(x_1, x_2, \ldots, x_N)$ that make up the matrix $\mathbf{X}$.

Using optimal experimental design methods, we can find the ideal set of points to **minimize the variance of our results** and **reduce the amount of sampling** needed before ever running the experiment.

In our work, we investigate two optimal experimental design methods, D-Optimal and C-Optimal, and compare them against the established linear and logarithmic spacing methods.

Current Methods of Evaluating 1/Λ

To estimate the error-scaling parameter Λ^{-1}, Deltakit evaluates Λ^{-1} at **several values of a chosen noise parameter** and fits a polynomial curve to the resulting data.

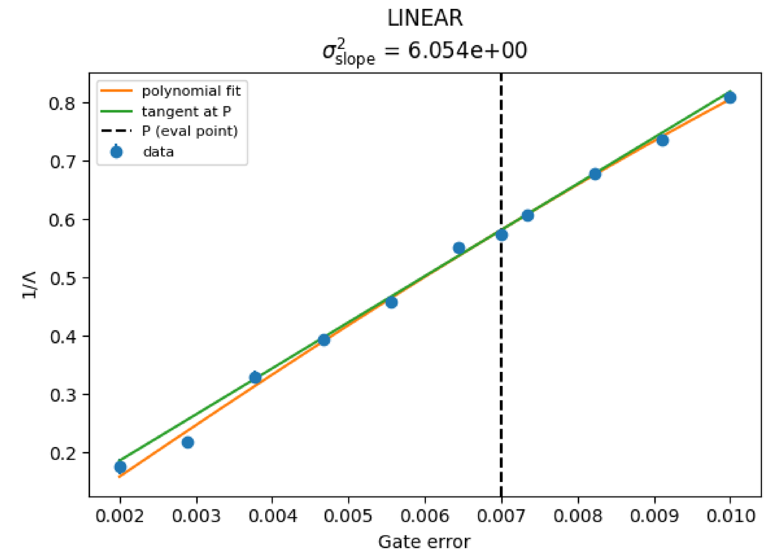
The gradient of this fitted curve at the target noise value P is then used to estimate **how strongly this noise source contributes to the total error budget**.

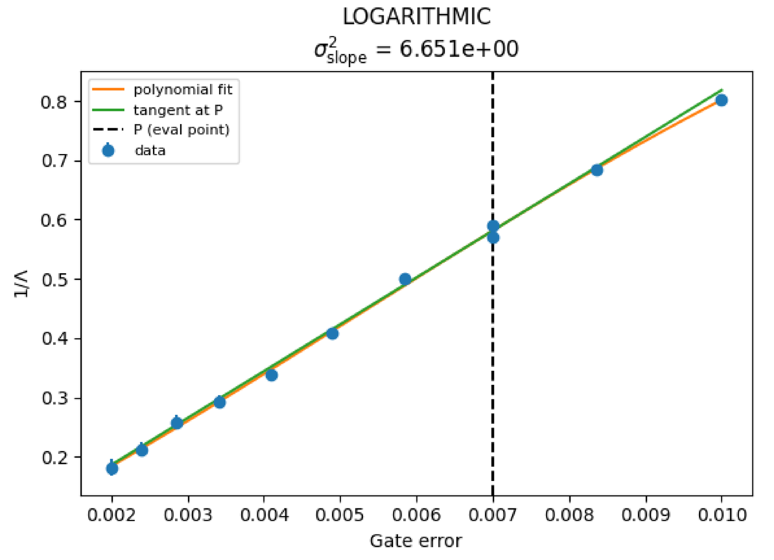
Linear Selection of Points:

- Noise values are selected at evenly spaced intervals.

Logarithmic Selection of Points:

- Noise values are spaced evenly on a logarithmic scale.





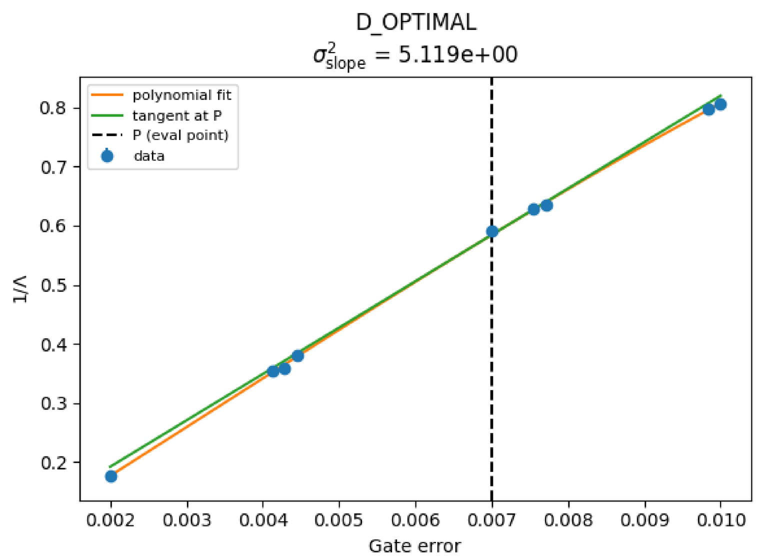
Implementing Two ODE's

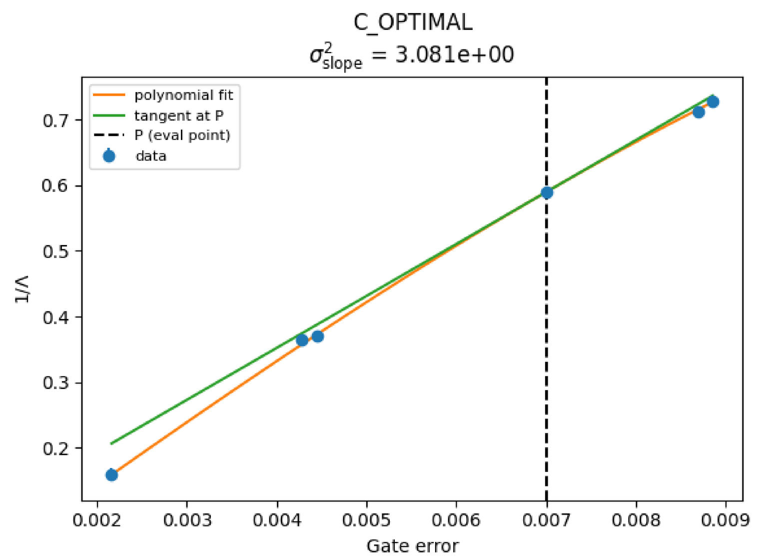
D-Optimal:

- Chooses noise parameter points that make the **overall polynomial fit for Λ^{-1} as stable as possible**. Selects the most informative simulation points rather than simply spacing them evenly.

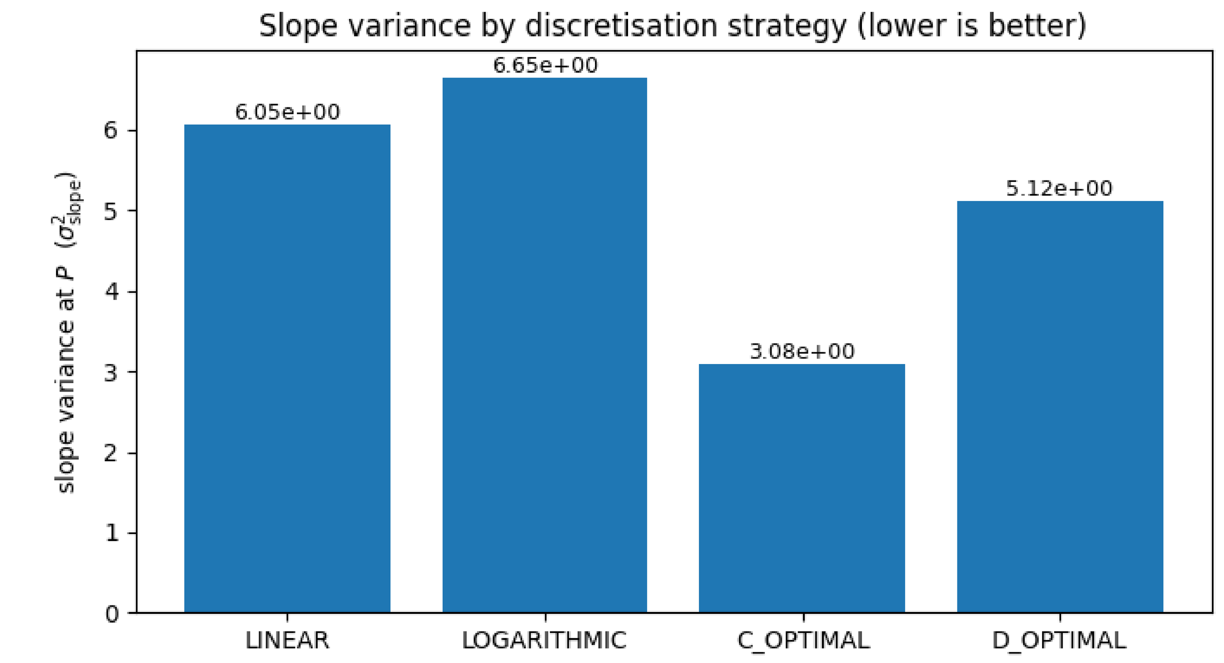
C-Optimal:

- Targets **one specific linear combination of coefficients**; in this case, that quantity is the slope of Λ^{-1} at the chosen physical noise value P.
- Instead of improving the whole curve equally, it **chooses points that most directly reduce uncertainty in this gradient estimate.**





Both methods **improve upon the linear and logarithmic point selections in reducing the variance of the slope of Λ^{-1}.**
Additionally, **fewer points are used in calculating this gradient**.



References

Riverlane. Error budgeting for quantum error correction. url: https://deltakit-docs . riverlane.com/en/stable/guide/error_budgeting.html.
Nielsen, Michael A., and Isaac L. Chuang. Quantum computation and quantum information. Cambridge university press, 2010.
Atkinson, Anthony, et al. Optimum Experimental Designs, with SAS, Oxford University Press, Incorporated, 2007. ProQuest Ebook Central, https://ebookcentral.proquest.com/lib/washington/detail.action?docID=415611.

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